## Peter Cheung, v1.0

In order to design the ping-pong ball "kicking" mechanism, we need to consider the drag creating when the ball travel through air at a given velocity.

The ping-pong ball has a weight of 2.7g and a diameter of 4cm. The drag Fd of a sphere through air is given by the equation:

$$F_d = \frac{1}{2} C_d \rho A v^2$$

where  $C_d$  is the drag coefficient and is around 0.5 for a sphere,  $\rho$  is air density which is 1.2kg/m<sup>3</sup>. For the ping-pong ball,  $A = \pi r^2 = 0.00126 \text{ m}^2$ .

Therefore  $F_d = 0.000378v^2 = K_d v^2$ .

If you use a spring, a rubber band or some other energy storage method, you can measure the force that is exerted on the ping ball, Fs. I have a digital spring loaded weighing machine in the Lab for you to measure the force of your mechanism.z

Now we can combine this together. At the rest position, just before you release the trigger, the force exerted on the ball  $F_b = m_b \frac{dv}{dt} = F_s - F_d = F_s - K_d v^2$ . We will for now assume that Fs is constant throughout the ball's travel.

Unfortunately, this is a first-order non-linear differential equation and it is beyond the scope of your maths course.

However, the differential equation of the general form of:

Riccati's equation:

$$y'(t) = b y(t)^2 + a$$

has a known solution:

$$y(t) = \frac{\sqrt{a} \tan(\sqrt{a} \sqrt{b} c_1 + \sqrt{a} \sqrt{b} t)}{\sqrt{b}}$$

## Simplification

Instead of dealing with such this difficult maths problem, you can make some engineering estimates.

Assuming that the ball travels through air at an *average* velocity of 10m/s (36km/hr) when shooting a penalty, you can calculate the average force required on the ball, and hence estimate the tension (force) you need on your storage device.

**In conclusion,** you can use engineering analysis you have learned this year to help you design the actual mechanism that you design. Such analysis helps you to go beyond just trial-and-error.